# Storage-ring free-electron-laser amplifiers and the microwave instability

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We discuss the storage-ring longitudinal dynamics by including the contributions from a free-electron-laser amplifier and from the microwave instability. We show that the two effects exhibit a mutual interplay that leads, under certain conditions, to the inhibition of the start up of the microwave instability and/or to a reduction of the electron-beam heating due to the free-electron laser. The agreement with the experimental results is also discussed. [S1063-651X(97)09001-6]

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Storage ring (SR) free-electron lasers (FELs) have displayed a fairly intriguing dynamical behavior. The models proposed so far have reproduced satisfactorily well the relevant micro- and macrotemporal aspects [1] and have also suggested the existence of a chaotic dynamics [2].

The peculiar aspects displayed by the SR FEL evolution are associated with the interplay between the FEL-induced heating and the damping due to synchrotron radiation emission. The characteristic times governing the longitudinal damping cannot be considered independent of the FEL itself. It has indeed been shown that the damping times of the multipole oscillation modes may increase or decrease according to the FEL operating conditions [3]. In the region of positive gain slope, the damping times increase. This is expected, in fact, for higher energies, with respect to the synchronous particle, the beam is emitting (in the average) more (stimulated) radiation and the energy shift is thereby reduced.

The fact that the FEL may control the longitudinal damping is suggestive of the possibility that the FEL interaction may affect or even counteract some of the longitudinal electron-beam instabilities. In this paper we will discuss the mutual interplay between a FEL amplifier and the so-called microwave instability (see Dattoli and Renieri's work in Ref. [1]), responsible for the anomalous bunch lengthening. According to this effect, the equilibrium electron-beam parameters are far from those predicted by the single-particle theory [4]. The phenomenon has been explained as the occurrence of a longitudinal instability due to the coupling of the electrons with the SR environment, like electrodes and spurious cavities. The items responsible for this instability are resonating at high frequencies (typically in the microwave region) and exhibit low $Q$. The relevant wavelengths are therefore short compared to the electron-bunch length and the instability is therefore a single-bunch effect without any machine turn by turn memory.

Previous investigations [5,6] have been able to account for the main consequences, i.e., instability of the equilibrium configuration and evolution of the bunch towards a new configuration with consequent increase of the bunch length and of the energy spread. In the following we will see how a superimposed FEL amplifier modifies the dynamics of the microwave instability.

The model equations describing the individual particle evolution, after each machine turn $n$, are $[(\overline{\varepsilon,z})\equiv(\varepsilon_{n+1},z_{n+1})$ and $(\varepsilon,z)\equiv(\varepsilon_n,z_n)]$

$$\overline{z}=z+c\alpha_c T_0\overline{\varepsilon}, \tag{1a}$$

$$\begin{aligned}\overline{\varepsilon}=&\varepsilon-\frac{\omega_s^2T_0}{c\alpha_c}z-2\frac{T_0}{\tau_s}\varepsilon+2\sigma_\varepsilon^0\sqrt{\frac{T_0}{\tau_s}}\,r\\&+\rho\omega_sT_0\sigma_\varepsilon^0\frac{1}{\mathcal{N}}\sum_{k=1}^{\mathcal{N}}\exp\left[-\frac{\mu\sigma_p}{\sigma_z^0}(z-z_k)\right]\vartheta(z-z_k)\\&+\frac{1}{N}\sqrt{\frac{T_0W}{2\tau_s}}\,\mathrm{sinc}[2\pi N(\varepsilon+\varepsilon_0)]\\&\times\sin[\varphi-2\pi N(\varepsilon+\varepsilon_0)],\end{aligned} \tag{1b}$$

$$\varphi=\varphi_0-2\pi\frac{z}{\lambda}\ (\mathrm{mod}\ 2\pi). \tag{1c}$$

The symbols are listed in Table I. We have denoted by $z$ the longitudinal coordinate and by $\varepsilon$ the normalized energy.

The above difference equations are derived under the assumption of small-amplitude synchrotron motion. The last two terms in Eq. (1b) account for the microwave instability and for the FEL interaction, respectively, treated under the assumption of impulsive approximation.

The physical meaning of the microwave instability contribution is that of a force term due to a mean field, excited by all the electrons before the $j$th electron and experienced by the $j$th electron itself. This field is characterized by a strength $\rho$ and by a decay constant $\mu\sigma_p/\sigma_z^0$. This is one of the possible ways of modeling the microwave instability and it is based on the assumption that this instability is provided by a single capacitive spurious resonating item [5]. The FEL interaction term is responsible for the well-known beam heating effect [7].

To understand the interplay between the two contributions we focus our attention on the behavior of the quantities

$$\Delta_\varepsilon(t)=\frac{\sigma_\varepsilon^2(t)-(\sigma_\varepsilon^0)^2}{2},\quad \widetilde{\Delta}_\varepsilon=\frac{1}{n_T}\sum_{j=1}^{n_T}\Delta_\varepsilon(j). \tag{2}$$

The first measures the deviation at the time $t$ of the rms energy spread from the initial equilibrium value. The second is the average of $\Delta_\varepsilon(t)$ on a number of turns corresponding to $t(n_T=t/T_0)$. We have adopted this definition to provide a

TABLE I. Symbols.

| |
|---|
| $z$ is the longitudinal displacement of the particle from the synchronous one (if $z<0$ the particle is passing ahead of the bunch) |
| $\varepsilon=(E-E_0)/E_0$ is the relative shift of the particle energy ($E$) from the synchronous energy ($E_0$) |
| $\omega_s$ is the synchronous frequency |
| $\alpha_c$ is the momentum compaction |
| $T_0$ is the revolution period |
| $\tau_s$ is the longitudinal radiation dampling time |
| $\sigma_\varepsilon^0$ is the equilibrium relative energy spread, without FEL and anomalous bunch lengthening |
| $\sigma_z^0$ is the equilibrium rms bunch length, without FEL and anomalous bunch lengthening |
| $\mathcal{N}$ is the number of electrons in the bunch |
| $r$ is the random number between 0 and 1 |
| $D$ is the radiation noise diffusion coefficient |
| $\lambda_u$ is the laser radiation wavelength |
| $k$ is the UM parameter |
| $(\Delta\omega/\omega)_0=1/2N$, the homogeneous linewidth ($N$ is the number of UM periods) |
| $\sqrt{\langle\delta\varepsilon^2\rangle}=\sqrt{A\ \mathrm{sinc}\left(\frac{2(\varepsilon+\varepsilon_0)}{(\Delta\omega/\omega)_0}\right)}$ is the energy spread induced by the FEL interaction |
| $A=\frac{\pi r_0}{m_0c^3}\lambda^2\left(\frac{\Delta\omega}{\omega}\right)_0\frac{k^2}{(1+k_2^2)^2}[f_b(\xi)]^2 I$ |
| $f_b(\xi)=J_0(\xi)-J_1(\xi),\quad \xi=\frac{1}{4}\frac{k^2}{1+k^2/2}$ |
| $I$ is the laser power density |
| $\varepsilon_0=(E_0-E^*)/E_0$, |
| $\nu_0=4\pi N\varepsilon_0$ |
| $E^*=m_0c^2\sqrt{\frac{\lambda_u(1+k^2/2)}{2\lambda}}$ is the FEL resonant energy |
| $\lambda$ is the laser radiation wavelength |
| $W=\frac{\tau_s}{T}\frac{\pi r_0}{m_0c^2}\left(\frac{\Delta\omega}{\omega}\right)_0^{-4}\frac{\lambda^2}{2c}\frac{k^2}{(1+k^2/2)^2}[f_b(\xi)]^2\frac{dP_L}{dS}$ |
| $\frac{dP_L}{dS}=\frac{\overline{x}}{\mu}I_s$ |
| $I_s\ (\mathrm{MW/cm^2})=6.9\times10^2(\gamma/N)^4[\lambda_u(\mathrm{cm})Kf_b(\xi)]^{-2}$ |
| $\mu=(0.433/N)^2\frac{\pi}{8}\tau_s/T(\sigma_\varepsilon^0)^2$ |

TABLE II. Numerical values used for the simulation.

| |
|---|
| $\sigma_z^0=2.7$ cm |
| $\sigma_\varepsilon^0=8\times10^{-4}$ |
| $T_0=2.4\times10^{-7}$ s |
| $\omega_s=c\alpha_c\frac{\sigma_\varepsilon^0}{\sigma_z^0}=1.2\times10^5\ \mathrm{s}^{-1}$ |
| $\tau_s=1.5\times10^{-3}$ s |
| $\alpha_c=1.4\times10^{-2}$ |
| $N=100$ |
| $W=1.12$ |
| $\underline{\mu\sigma_p}=5$ |
| $\overline{x}=28$ |

value averaged on the synchrotron oscillations. Other definitions, involving shorter intervals of time, can be adopted, and we have checked that they yield the same physical informations.

Any deviation from the equilibrium, due to the FEL or to the anomalous bunch lengthening, is expected to provide both $\Delta_\varepsilon(t)$ and $\widetilde{\Delta}_\varepsilon$ differing from zero. To study the system evolution we have performed a numerical simulation that uses the parameters of Table II as input values. The simulation uses $9\times10^3$ stochastic particles per bunch and is based on a Monte Carlo procedure whose stability has been checked by varying the number of stochastic particles.

In Fig. 1 we show $\widetilde{\Delta}_\varepsilon$ averaged on two damping times, for which the system has already reached a stationary configuration. The continuous line refers to the case in which the FEL and the microwave instability are switched on (i.e., $W\neq0$, $\rho\neq0$), the dashed line to the case in which only the second effect is active. When the FEL is switched off ($W=0$), the microwave instability occurs after the threshold value $\rho^*=24$. When the FEL is superimposed, the final result is not the incoherent sum of both contributions. In the case of Fig. 1 the FEL produces the usual heating as in the absence of microwave instability even for $\rho>\rho^*$. The occurrence of anomalous bunch lengthening effects is shifted towards larger $\rho$ values and for $50<\rho<120$ the induced energy spread is smaller than that predicted by the sum of the two separated contributions. At very large $\rho$ values the anomalous bunch lengthening dominates and the system seems to be insensitive to FEL amplifier perturbations.

At zero gain ($\nu_0=0$) [Fig. 1(b)], the situation is more interesting. We can divide the $\rho$ values into three regions: below the threshold ($\rho<\rho^*$) the system is sensitive to FEL only, above the threshold the combined contributions is such that the usual FEL heating is reduced, and when $\rho$ is very large the system is insensitive to the FEL and the anomalous bunch lengthening dominates. It is clear that for $W<1.12$ the above phenomenology is shifted towards smaller $\rho$ values and vice versa for $W>1.12$. The actual $W$ values correspond to the intracavity power of the SuperACO experiment [8].

The previous conclusions are confirmed by Fig. 2, which provides the dynamical behavior of $\Delta_\varepsilon$ up to two damping times. Figure 2(a) shows that for $\rho=50$, $\nu_0=2.6$, and $W=1.12$, the system evolution is dominated by the FEL and is insensitive to the microwave instability. In Fig. 2(b), relevant to the system evolution for $\rho=100$, $\nu_0=0$, and $W=1.12$, the presence of the anomalous bunch lengthening partially ''cools'' the FEL heating. The above results can be

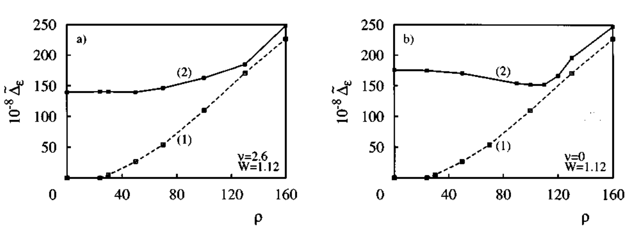


FIG. 1. (a) $\widetilde{\Delta}_\varepsilon$ averaged on two damping times (3 ms) vs $\rho$. Curve (1) the FEL is not active. Curve (2) the FEL is active; (b) Same as (a) but $\nu_0=0$.

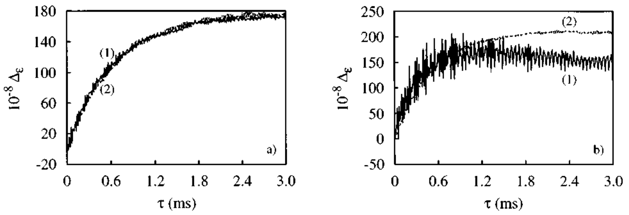


FIG. 2. (a) Evolution of $\Delta_\varepsilon$ vs time, $\rho=50$, $W=1.12$, $\nu_0=2.6$. (1) Both contributions are included; (2) only FEL is active; (b) same as (a) but $\nu_0=0$, $\rho=100$.

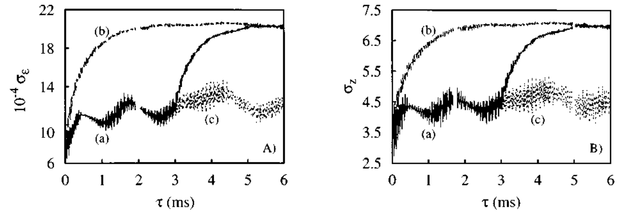


FIG. 3. (A) Relative energy spread vs time $\rho=50$, $W=1.12$, and $\nu_0=2.6$. $a$, the FEL is switched on after two damping times; in the first part only the microwave instability is active. $b$, the FEL and the microwave instability are simultaneously switched on at $t=0$. $c$, the system evolves up to 6 ms with the only influence of the microwave instability. (B) rms bunch length vs time as (A).

understood as the occurrence of a mechanism of mutual feedback between the FEL and the anomalous bunch lengthening. The presence of the FEL inhibits, within certain limits, the microwave instability. Furthermore, the partial cooling of the FEL heating should be understood by taking into account also the results of Ref. [3], which have shown that, in the positive gain slope region ($-2.6<\nu_0<2.6$), the longitudinal damping times increase. We have indeed checked the above results at $\nu_0=1.3$ and we have noted the presence of the cooling effect, which is less pronounced than at $\nu_0=0$, where the gain slope is maximum. To further support the above conclusions we have considered the negative gain slope region, where the damping times increase. For these $\nu_0$ values, and in the region in which the two effects cooperate, we have noted a kind of anticooling, i.e., the FEL heating effect is larger than that predicted by the ordinary theory.

It can be argued against the above analysis by noting that our examples are relevant to the case in which the system starts from the equilibrium situation and the FEL and the microwave instability are simultaneously switched on. In reality the FEL is switched on some time after the anomalous bunch lengthening, occurring at the switching on of the SR. To clarify this point we have performed a numerical experiment summarized in Fig. 3, relevant to the time evolution of the rms energy spread and rms bunch length. In the first 3 ms the system develops under the effect of the microwave instability ($\rho=50$). The evolution is rather noisy; when the FEL is superimposed ($\nu_0=2.6$, $W=1.12$), both $\sigma_z$ and $\sigma_\varepsilon$ increase and evolve with a significant reduction of the fluctuations. According to the above experiment, the stationary values of the energy spread and of the bunch length do not depend on the switching on time of the FEL interaction.

The results obtained in this paper confirm some experimental evidence [8,9]. It has indeed been observed as a systematic effect that the onset of the laser damps the synchrotron modes of oscillation and stabilizes the electron bunches by acting as a feedback system on them. In addition, even though the FEL interaction is known to induce ''bunch heating'' with a consequent increase of energy spread and bunch length, other situations occur and no modification of the bunch length or even bunch shortening have been observed. In conclusion, the present analysis confirms that the FEL may be a stabilizing element of the electron bunches.

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[1] P. Elleaume, IEEE J. Quantum Electron. **21**, 1012 (1985); V. N. Litvinenko, B. Nurnham, and J. M. J. Madey, Nucl. Instrum. Methods Phys. Res. Sect. A **358**, 369 (1995); G. Dattoli and A. Renieri, Nucl. Instrum. Methods Phys. Res. Sect. A **375**, 1 (1996).
[2] M. Billardon Phys. Rev. Lett. **65**, 713 (1990).
[3] G. Dattoli, L. Giannessi, and A. Renieri, Opt. Commun. **123**, 353 (1996).
[4] M. Sands, Stanford Linear Accelerator Report Report No. SLAC-121, 1970 (unpublished).
[5] A. Renieri, Laboratori Nazionali di Frascati Report No. LNF-75/11(R) 1975 (unpublished).
[6] A. M. Sessler, Lawrence Berkeley Laboratory Report No. 28, 1973 (unpublished).
[7] A. Renieri, Nuovo Cimento B **53**, 160 (1979); G. Dattoli and A. Renieri, Nuovo Cimento B **59**, 1 (1980).
[8] M. E. Couprie, M. Velghe, R. Prazers, D. Jaroszynski, and M. Billardon, Phys. Rev. A **44**, 1301 (1991).
[9] M. E. Couprie, D. Garzella, C. Monet-Descomby, M. Sommer, and M. Billardon, in *Proceedings of the Third Particle Accelerator Conference, Berlin, 1992*, edited by H. Henke, H. Homeyer, and Ch. Petit-Jean-Genaz (Éditions Frontières, Berlin, 1992), p. 623.